Optical Properties Exam Question #1: Lorentz Oscillator Model (10 pts)

In the Lorentz Oscillator Model, we consider the motion of a charged particle bound to a fixed core of opposite charge.

a) (2pts)Draw the frequency dependent response of a single negatively changed particle (with a single resonant frequency) to an applied harmonically oscillating driving field. You are welcome to draw either ε or χ. Draw the real and imaginary parts separately. (3pts)

This is something we went over a lot in class and should be freebee. The often seen real and imaginary parts are the Lorentzian function (imaginary) and the bimodal Lorentzian multiplied by ($\omega_0 - \omega$). Details matter here, so I would like to see correspondence between widths, axes labels, etc.

b) (2pts) Consider a material with a single resonance at 3μm (near infrared). Based on the graphs you drew above, circle the correct statement below (n is the real portion of the index of refraction):

$n_{800nm} > n_{700nm}$, $n_{800nm} < n_{700nm}$, Can't tell

In drawing the real part of the dielectric constant $\varepsilon=n^2$, like in above, and placing the resonance as well as 800nm and 700nm on the same graph, it is clear that the $n^2(800nm) < n^2(700nm)$ and hence the middle choice is the correct one.

c) (4pts) Draw the corresponding dielectric response for a free electron gas. Do so for both the situation of no damping and the case with damping. The precise details of the drawing can be left vague, but the general response should be evident

The key element to the free electron response is that the real part can be negative:

$$\varepsilon = n^2 = 1 - \frac{\omega_P^2}{\omega^2}$$

The line should therefore cross zero at ω_p and be large and negative for frequencies below, asymptoting to 1 for frequencies above. The case with damping will now involve an imaginary value will starts at high values at low frequencies and approaches 0 as the frequency is increased.

d) (2pts)For a metal modeled with a free electron response, predict the normal incidence reflectivity for light at a frequency lower than the plasma resonance.

For the case of light at frequencies below the plasma frequency, the real component of the dielectric constant is large and negative. Therefore when taking a square root, we see that the complex index of refraction to an approximation is only comprised of the absorption constant k from N= n+ik. In the equation for normal incidence reflection,

$$R = \left|\frac{n + ik - 1}{n + ik + 1}\right|^2 \Rightarrow \left|\frac{ik - 1}{ik + 1}\right|^2 \Rightarrow \left|\frac{k}{k}\right|^2 = 1$$

Where I have made use of the fact that k >> 1.

Optical Properties Exam Question #2: Absorption of Light in Materials (10pts)

a) Starting from the equation for a wave propagating in a material $E = E_0 e^{i(kx-\omega t)}$, show that the inclusion of a complex component to the Index of Refraction leads to absorption (2pts).

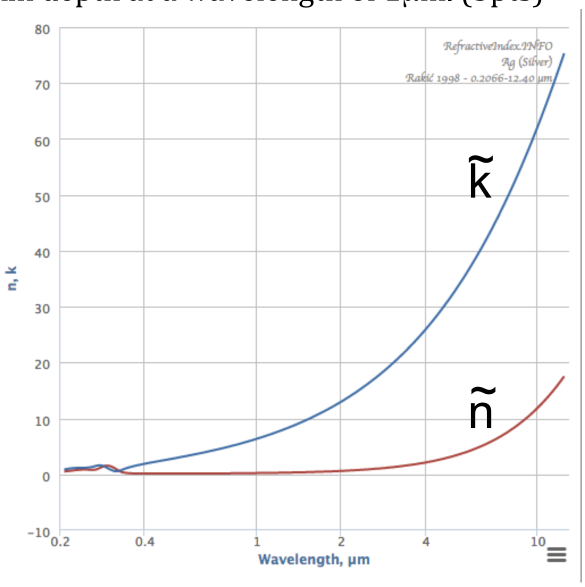
The solution should simply be to substitute $k \to k\tilde{N}$ where $N = \tilde{n} + i\tilde{k}$. This is following the Dresselhaus notation which we agreed to in class.

b) Absorption of light occurs over a lengthscale called the *skin depth*, where the intensity falls off to 1/e from its initial value. From the expression you arrived at above, find the skin depth. (3pts)

From the above expression, a direct plug in shows that the electric field intensity falls like $e^{-k\tilde{k}z}$ which gives the intensity dropping as $e^{-2k\tilde{k}z}$. The skin depth is then

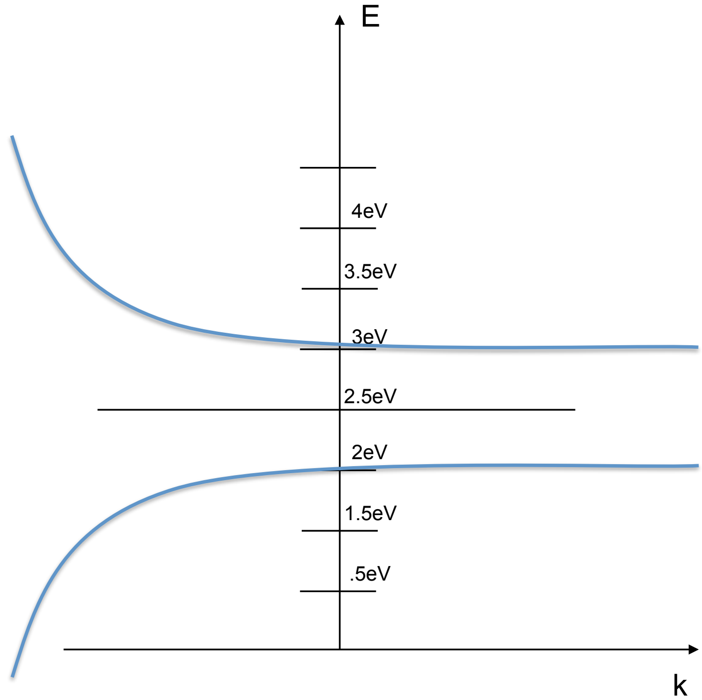
$$\delta = \frac{1}{2k\tilde{k}}$$

c) Estimate the skin depth at a wavelength of 1μm. (3pts)



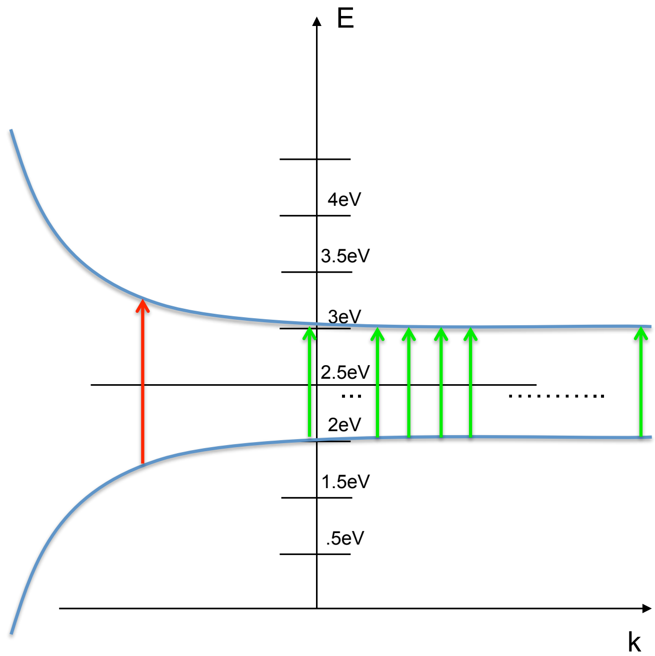
Using the equation above, one first must change the wavelength into wavevector, then estimate the value at 1μm (I would say 6 or 7) and then calculate the skin depth from above.

d) Consider the contrived band structure shown below (the 2.5eV line corresponds to the Fermi Energy) (2pts):



On this graph draw an interband transition with a photon of 1.5eV. Similarly, draw (all possible) interband transitions at 1eV.

The solution should be roughly the following. Importantly, all arrows should be vertical, and there should be only one arrow for 1.5eV and several or many arrows for 1eV.



Optical Properties Exam Question #3: Nonlinear Response (10pts)

a) Write the full expansion of material Polarization to include up to third order effects (1pt).

The answer is $P = \chi^{(1)}E + \chi^{(2)}EE + \chi^{(3)}EEE + \ldots$ or a corresponding version with ε_0 attached

b) The polarization acts as a source term for the wave equation and leads to generation of new frequencies of light (the second harmonic in this case). Discuss in general terms how the intensity of generated light scales with the intensity of the input field. (2pts).

Since the polarization scales as the intensity of the input field, and this acts as a source for the electric field of the generated beam, then the intensity of the generated beam scales as I^2 of the input beam.

c) Correspondingly, how does the output intensity of a third harmonic generation process scale with input intensity? (2pt)

Either by explicit calculation or by deduction, the intensity of the output beam scales as I^3.